Exercise 24

Find y' and y''.

$$y = \frac{\ln x}{1 + \ln x}$$

Solution

Take the derivative of the function.

$$\begin{aligned}
y' &= \frac{d}{dx}\left(\frac{\ln x}{1+\ln x}\right) \\
&= \frac{\left[\frac{d}{dx}(\ln x)\right](1+\ln x) - \left[\frac{d}{dx}(1+\ln x)\right](\ln x)}{(1+\ln x)^2} \\
&= \frac{\left(\frac{1}{x}\right)(1+\ln x) - \left(\frac{1}{x}\right)(\ln x)}{(1+\ln x)^2} \\
&= \frac{1}{x(1+\ln x)^2}
\end{aligned}$$

Take another derivative.

$$\begin{aligned}
y'' &= \frac{d}{dx}(y') \\
&= \frac{d}{dx}\left[\frac{1}{x(1+\ln x)^2}\right] \\
&= \frac{\left[\frac{d}{dx}(1)\right]x(1+\ln x)^2 - \left\{\frac{d}{dx}[x(1+\ln x)^2]\right\}(1)}{[x(1+\ln x)^2]^2} \\
&= \frac{(0)x(1+\ln x)^2 - \left\{\left[\frac{d}{dx}(x)\right](1+\ln x)^2 + x\left[\frac{d}{dx}(1+\ln x)^2\right]\right\}}{[x(1+\ln x)^2]^2} \\
&= \frac{-\left\{(1)(1+\ln x)^2 + x\left[2(1+\ln x)\cdot\frac{d}{dx}(1+\ln x)\right]\right\}}{x^2(1+\ln x)^4} \\
&= -\frac{(1+\ln x)^2 + x\left[2(1+\ln x)\cdot\left(\frac{1}{x}\right)\right]}{x^2(1+\ln x)^4} \\
&= -\frac{(1+\ln x)^2 + 2(1+\ln x)}{x^2(1+\ln x)^4} \\
&= -\frac{(1+\ln x)+2}{x^2(1+\ln x)^3} \\
&= -\frac{3+\ln x}{x^2(1+\ln x)^3}
\end{aligned}$$